## Exercise 13

Compute  $\mathbf{u} + \mathbf{v}$ ,  $\mathbf{u} \cdot \mathbf{v}$ ,  $\|\mathbf{u}\|$ ,  $\|\mathbf{v}\|$ , and  $\mathbf{u} \times \mathbf{v}$ , where  $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ ,  $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ .

## Solution

Write each of the vectors as

$$\mathbf{u} = (1, -2, 1)$$
  
 $\mathbf{v} = (2, -1, 2).$ 

Then

$$\begin{aligned} \|\mathbf{u}\| &= \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6} \\ \|\mathbf{v}\| &= \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3 \\ \mathbf{u} + \mathbf{v} &= (1, -2, 1) + (2, -1, 2) = (3, -3, 3) \\ \mathbf{u} \cdot \mathbf{v} &= (1, -2, 1) \cdot (2, -1, 2) = (1)(2) + (-2)(-1) + (1)(2) = 6 \\ \mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & -2 & 1 \\ 2 & -1 & 2 \end{vmatrix} = (-4 + 1)\hat{\mathbf{x}} - (2 - 2)\hat{\mathbf{y}} + (-1 + 4)\hat{\mathbf{z}} = -3\hat{\mathbf{x}} + 3\hat{\mathbf{z}} = (-3, 0, 3). \end{aligned}$$