## Exercise 13

Compute $\mathbf{u}+\mathbf{v}, \mathbf{u} \cdot \mathbf{v},\|\mathbf{u}\|,\|\mathbf{v}\|$, and $\mathbf{u} \times \mathbf{v}$, where $\mathbf{u}=\mathbf{i}-2 \mathbf{j}+\mathbf{k}, \mathbf{v}=2 \mathbf{i}-\mathbf{j}+2 \mathbf{k}$.

## Solution

Write each of the vectors as

$$
\begin{aligned}
& \mathbf{u}=(1,-2,1) \\
& \mathbf{v}=(2,-1,2) .
\end{aligned}
$$

Then

$$
\begin{aligned}
\|\mathbf{u}\| & =\sqrt{1^{2}+(-2)^{2}+1^{2}}=\sqrt{6} \\
\|\mathbf{v}\| & =\sqrt{2^{2}+(-1)^{2}+2^{2}}=\sqrt{9}=3 \\
\mathbf{u}+\mathbf{v} & =(1,-2,1)+(2,-1,2)=(3,-3,3) \\
\mathbf{u} \cdot \mathbf{v} & =(1,-2,1) \cdot(2,-1,2)=(1)(2)+(-2)(-1)+(1)(2)=6 \\
\mathbf{u} \times \mathbf{v} & =\left|\begin{array}{ccc}
\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\
1 & -2 & 1 \\
2 & -1 & 2
\end{array}\right|=(-4+1) \hat{\mathbf{x}}-(2-2) \hat{\mathbf{y}}+(-1+4) \hat{\mathbf{z}}=-3 \hat{\mathbf{x}}+3 \hat{\mathbf{z}}=(-3,0,3) .
\end{aligned}
$$

